Lecture 5.

Higgs effect in abelian Gauge Theory.

Plan.

1. Higgs mechanism in scalar QED by diagrams.

1.1. Gauge symmetry and massless photon.

Recall the QED Lagrangian

$$L_{QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \bar{\psi}(\imath\partial_\mu\gamma^\mu - eA_\mu\gamma^\mu - m)\psi,$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (1)$$

It is invariant under the gauge symmetry

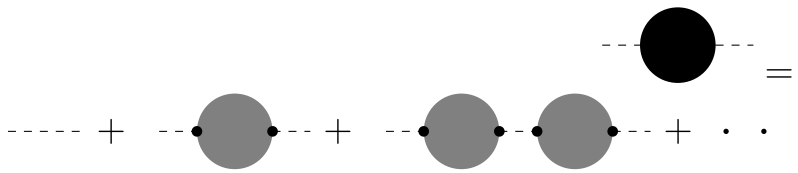
$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x), \psi(x) \to \exp{[\imath\alpha(x)]}\psi(x) \qquad (2)$$

We are interested in total propagator for gauge field

$$D_{\mu\nu}(k) =< \Omega|TA_\mu(k)A_\nu(-k)|\Omega >=$$

$$D^0_{\mu\nu}(k) + D^0_{\mu\lambda}(k)P^{\lambda\sigma}(k)D^0_{\sigma\nu}(k) + D^0_{\mu\lambda}(k)P^{\lambda\sigma}(k)D^0_{\sigma\rho}(k)P^{\rho\tau}(k)D^0_{\tau\nu}(k) + ... (3)$$

which means the following sum of diagrams



(4)

where

$$D^0_{\mu\nu}(k) = \frac{-\imath}{k^2}(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \equiv \frac{-\imath}{k^2}(g_\perp)_{\mu\nu} \tag{5}$$

is a free propagator and $P^{\mu\nu}(k)$ is a polarization operator which is given by a sum of amputated diagrams which can not be devided by cutting one of internal lines. Ward identity (see Appendix A)

$$k_\mu P^{\mu\nu}(k) = 0 \tag{6}$$

and Lorentz invariance constraint

$$P^{\mu\nu}(k) = A(k^2)g^{\mu\nu} + B(k^2)k^\mu k^\nu \tag{7}$$

allow to write

$$P^{\mu\nu}(k) = \imath(k^2 g^{\mu\nu} - k^\mu k^\nu)p(k^2) = \imath k^2 g^{\mu\nu}_\perp p(k^2) \tag{8}$$

Then we can perform the summation in (**??**) and obtain

$$D_{\mu\nu}(k) = \frac{-\imath}{k^2}(g_\perp)_{\mu\nu} + \frac{-\imath}{k^2}(g_\perp)_{\mu\nu}p(k^2) + \frac{-\imath}{k^2}(g_\perp)_{\mu\nu}p^2(k^2) + ... = \frac{-\imath(g_\perp)_{\mu\nu}}{k^2(1-p(k^2))} \tag{9}$$

where the relation

$$(g_\perp)_{\mu\nu}g^{\nu\lambda}(g_\perp)_{\lambda\rho} = (g_\perp)_{\mu\rho} \tag{10}$$

has been used.

We see that

if $p(k^2)$ is regular function at $k^2 = 0$ the photon stay massless.

Indeed, in the limit $k^2 \to 0$

$$D_{\mu\nu}(k) \to \frac{-\imath (g_{\perp})_{\mu\nu}}{k^2} Z_3,\ Z_3 = \frac{1}{1 - p(0)} \tag{11}$$

we have a massless photon with renormalized field strenght of $A_{mu}(x)$.

1.2. EM field interacting with scalar field and Higgs effect.

Let us consider the Lagrangian

$$L(A, \psi, \Phi) = L_{EM} + |D_\mu \Phi|^2 + \mu^2 \Phi^* \Phi - \frac{\lambda}{4} (\Phi^* \Phi)^2,$$
$$L_{EM} = -\frac{1}{4} (F_{\mu\nu})^2,\ D_\mu = \partial_\mu + \imath e A_\mu \tag{12}$$

Under the gauge transformation we have

$$\Phi(x) \to \exp[\imath \alpha(x)] \Phi(x),\ \psi(x) \to \exp[\imath \alpha(x)] \psi(x),$$
$$A_\mu(x) \to A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x) \tag{13}$$

Let us expand the complex scalar field $\Phi(x)$ around the minima $\Phi_0 = \frac{\mu}{\sqrt{\lambda}}$ of its potential $V(\Phi) = -\frac{\mu^2}{2} \Phi^* \Phi + \frac{\lambda}{4} (\Phi^* \Phi)^2$

$$\Phi(x) = \Phi_0 + \frac{1}{\sqrt{2}} (\phi_1(x) + \imath \phi_2(x)),$$
$$V(\Phi) = -\frac{\mu^4}{4\lambda} + \frac{\mu^2}{2} \phi_1^2 + O(\phi^3) \tag{14}$$

Hence, ϕ_1 becomes massive while ϕ_2 is massless (Goldstone boson)

$$(m_1)^2 = \mu^2,\ (m_2)^2 = 0 \tag{15}$$

Then we can write

$$
\begin{aligned}
(D_\mu\Phi)^2 = (\frac{1}{\sqrt{2}}\partial_\mu\phi + \imath eA_\mu(\Phi_0 + \frac{\phi}{\sqrt{2}})(\frac{1}{\sqrt{2}}\partial^\mu\phi^* - \imath eA_\mu(\Phi_0 + \frac{\phi^*}{\sqrt{2}}) = \\
\frac{1}{2}((\partial_\mu\phi_1)^2 + (\partial_\mu\phi_2)^2) + e^2\Phi_0^2A_\mu A^\mu + \sqrt{2}e\Phi_0A^\mu\partial_\mu\phi_2 + \\
eA^\mu(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1) + \sqrt{2}e^2\phi_1A_\mu A^\mu + \frac{e^2}{2}(\phi_1^2 + \phi_2^2)A_\mu A^\mu = \\
\frac{1}{2}((\partial_\mu\phi_1)^2 + (\partial_\mu\phi_2)^2) + e^2\Phi_0^2A_\mu A^\mu + \sqrt{2}e\Phi_0A^\mu\partial_\mu\phi_2 + ...
\end{aligned}
\tag{16}
$$

we see the mass term $e^2\Phi_0^2A_\mu A^\mu$ appeared.

1.3. Higgs effect leads to a pole in photon polarization operator and generates photon mass.

Now we will demonstrate that due to the Godstone's boson exchange the gauge field A_μ becomes massive because of the polarization operator get pole.

From the quadratic terms we can read off the diagrams represented on fig. 1.

Vector boson propagator:

$$\frac{-\imath}{k^2}(g_\perp)_{\mu\nu} =$$

m - - - - - - - - - - - - - - - - - - n

(17)

Mass term vertex for the vector boson:

$$\imath\frac{m_A^2}{2}g_{\mu\nu} =$$

m - - - - - - - - - • - - - - - - - - - n (18)

Goldstone's boson propagator:

$$\frac{\imath}{k^2} =$$

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(19)

Vector boson Goldstone's boson transition

$$k_\mu m_A = \imath\sqrt{2}e\Phi_0(-\imath k_\mu) =$$

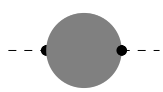
$$-k_\mu m_A = \imath\sqrt{2}e\Phi_0(\imath k_\mu) =$$

(20)

The leading oder contribution to the polarization operator for the vector boson is given by the Goldstone's boson exchange:

$$P^{\mu\nu}(k) = \imath m_A^2 g^{\mu\nu} - k^\mu m_A \frac{-\imath}{k^2}(-k^\nu m_A) = \imath m_A^2(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) = \imath m_A^2 g_\perp^{\mu\nu} \quad (21)$$

which can be represented by the diagram on fig.2.:

$$P^{\mu\nu}(k) = \imath m_A^2 g_\perp^{\mu\nu} =$$ m  n $=$

m n $+$ m n (22)

Notice that $P^{\mu\nu}(k)$ is transverse and has a pole at $k^2 = 0$.

Remark.

There is also a contribution to $P^{\mu\nu}(k)$ coming from the term $eA^\mu(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1)$ (see (**??**). But it is much less then the contribution from $\sqrt{2}e\Phi_0 A^\mu\partial_\mu\phi_2$ term because $\Phi_0 = \frac{\mu}{\sqrt{\lambda}}$ is bigger than the λ-perturbation theory series corrections.

One can summ up all the diagrams represented on fig.3. to obtain the total propagator of the field $A_\mu(x)$

$$D_{\mu\nu}(k) = \frac{-\imath}{k^2}(g_\perp)_{\mu\nu} + \frac{-\imath}{k^2}(g_\perp)_{\mu\lambda}P^{\lambda\sigma}(k)\frac{-\imath}{k^2}(g_\perp)_{\sigma\nu} + \frac{-\imath}{k^2}(g_\perp)_{\mu\lambda}P^{\lambda\sigma}(k)\frac{-\imath}{k^2}(g_\perp)_{\sigma\rho}P^{\rho\tau}(k)\frac{-\imath}{k^2}(g_\perp)_{\tau\nu} + ... = \frac{-\imath}{k^2}(g_\perp)_{\mu\nu} + \frac{-\imath}{k^2}(g_\perp)_{\mu\nu}\frac{m_A^2}{k^2} + \frac{-\imath}{k^2}(g_\perp)_{\mu\nu}\frac{m_A^4}{k^4} + ... = \frac{-\imath}{k^2}\frac{(g_\perp)_{\mu\nu}}{(1-\frac{m_A^2}{k^2})} = -\imath\frac{(g_\perp)_{\mu\nu}}{(k^2-m_A^2)} \tag{23}$$

Thus we see that gauge field becomes massive. From the other hand we can write

$$D_{\mu\nu}(k) = \frac{-\imath}{k^2-m_A^2}(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}) + \frac{-\imath k_\mu k_\nu}{m_A^2 k^2} \tag{24}$$

The first term is a propagator for the vector field $A_\mu(x)$ with the Lagrangian

$$-\frac{1}{4}F^2 - \frac{m_A^2}{2}A_\mu A^\mu \tag{25}$$

The equations of motion for A:

$$\partial^\mu F_{\mu\nu} + m_A^2 A_\nu = 0 \Rightarrow \partial^\nu A_\nu = 0 \Leftrightarrow k^\nu A_\nu(k) = 0 \tag{26}$$

Hence, A_μ has transverse polarization but the Lagrangian (**??**) gives non-renormalizable theory because of the term $\frac{k_\mu k_\nu}{m_A^2}$ in the propagator. Fortunately the last term from (**??**) cure the problem.

2. QED and Higgs effect in t'Hooft gauge fixing.

2.1. The Lagrangian and Higgs mechanism.

$$L = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\Phi|^2 - V(\Phi),$$
$$V(\Phi) = -\mu^2\Phi^*\Phi + \frac{\lambda}{2}(\Phi^*\Phi)^2,\ \Phi = \frac{1}{\sqrt{2}}(\Phi^1 + \imath\Phi^2)$$
$$D_\mu = \partial_\mu + \imath e A_\mu \tag{27}$$

Gauge transformations are given by

$$\delta\Phi^1 = -\alpha(x)\Phi^2,\ \delta\Phi^2 = \alpha(x)\Phi^1,\ \delta A_\mu = -\frac{1}{e}\partial_\mu\alpha \tag{28}$$

One can expand the potential $V(\Phi)$ around the vacuum:

$$\begin{aligned} \Phi_0 = \frac{1}{\sqrt{2}}v = \sqrt{\frac{2}{\lambda}}\mu,\ \frac{\partial V}{\partial\Phi^i}_{\Phi=\Phi_0} = 0, \\ \Phi(x) = \Phi_0 + \frac{1}{\sqrt{2}}(h(x) + \imath\phi(x)), \\ V(h,\phi) = -\frac{\mu^4}{2\lambda} + \mu^2 h^2(x) + \\ \frac{\lambda}{8}(4vh^3(x) + h^4(x)) + \frac{\lambda}{4}(2vh(x) + h^2(x))\phi^2(x) + \frac{\lambda}{8}\phi^4(x) \end{aligned} \tag{29}$$

Then we rewrite the covariant derivative term by the fields $h(x)$ and $\phi(x)$:

$$\begin{aligned} |D_\mu\Phi|^2 = \frac{1}{2}(\partial_\mu h - eA_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\phi + eA_\mu(v+h))^2 = \\ \frac{1}{2}[(\partial_\mu h)^2 + (\partial_\mu\phi)^2 + e^2v^2A_\mu A^\mu + 2evA^\mu\partial_\mu\phi(x) \\ -2e\phi(x)A^\mu(x)\partial_\mu h(x) + 2eh(x)A^\mu\partial_\mu\phi(x) \\ e^2(2vh(x) + h^2(x) + \phi^2(x))A_\mu A^\mu] \end{aligned} \tag{30}$$

The gauge transformations take the form

$$\delta h = -\alpha(x)h, \delta\phi = \alpha(x)(v+h), \delta A_\mu = -\frac{1}{e}\partial_\mu\alpha(x) \tag{31}$$

2.2. Paths integral and t'Hooft's gauge fixing.

Now we are interested in the paths integral

$$\begin{aligned} Z = \int[DA][Dh][D\phi]\exp[\imath\int d^4xL(A,h,\phi)] = \\ C\int[DA][Dh][D\phi]\exp[\imath\int d^4xL(A,h,\phi)]det(\frac{\delta G}{\delta\alpha})\delta(G(A,h,\phi)) \end{aligned} \tag{32}$$

where the gauge fixing has been introduced by a function $G(A,h,\phi) = 0$ and the result of integration over the gauge transformation orbits has been taken into account by the factor C. One also can change the gauge fixing conditions by a more general ones

$$G(A,h,\phi)=0 \to G(A,h,\phi)-\omega(x)=0 \tag{33}$$

and integrate out over all possible functions $\omega(x)$ weighted by $\exp[-\imath\int d^4x\frac{\omega(x)^2}{2\xi}]$:

$$\begin{aligned} Z = \\ N(\xi)\int[DA][Dh][D\phi][D\omega]\exp[-\imath\int d^4x\frac{\omega(x)^2}{2\xi}] \\ \exp[\imath\int d^4xL(A,h,\phi)]det(\frac{\delta G}{\delta\alpha})\delta(G(A,h,\phi)-\omega) = \\ N(\xi)\int[DA][Dh][D\phi]\exp[\imath\int d^4x(L(A,h,\phi)-\frac{1}{2\xi}G^2)]det(\frac{\delta G}{\delta\alpha}) \end{aligned} \tag{34}$$

In what follows we redefine $\frac{G^2}{\xi}\to G^2$.

There is a good choice of G:

$$G=\frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu-\xi ev\phi) \tag{35}$$

Notice that gauge condition depends on the Goldstone field Then

$$\begin{aligned} L(A,h,\phi)-\frac{1}{2}G^2 = \\ \frac{1}{2}A_\mu[g^{\mu\nu}(\partial)^2+(\frac{1}{\xi}-1)\partial^\mu\partial^\nu+(ev)^2]A_\nu \\ +\frac{1}{2}(\partial_\mu h)^2-\frac{1}{2}2\mu^2h^2+\frac{1}{2}(\partial_\mu\phi)^2-\frac{\xi}{2}(ev)^2\phi^2 \\ -e\phi A_\mu\partial^\mu h+ehA_\mu\partial^\mu\phi+\frac{e^2}{2}(2vh+h^2+\phi^2)A_\mu A^\mu+ \\ \frac{\mu^2}{2\lambda}-\frac{\lambda}{8}(4v+h)h^3-\frac{\lambda}{4}(2v+h)h\phi^2-\frac{\lambda}{8}\phi^4 = \\ \frac{1}{2}A_\mu[g^{\mu\nu}(\partial)^2+(\frac{1}{\xi}-1)\partial^\mu\partial^\nu+(ev)^2]A_\nu \\ +\frac{1}{2}(\partial_\mu h)^2-\frac{1}{2}2\mu^2h^2+\frac{1}{2}(\partial_\mu\phi)^2-\frac{\xi}{2}(ev)^2\phi^2+... \end{aligned} \tag{36}$$

We see that G^2 term contains the old gauge fixing condition $(\partial^\mu A_\mu)^2$.

The cross term in G^2 is engineered to cancel the term $A^\mu \partial_\mu \phi$ term in the Lagrangian (??).

One can see also that the Higgs field h, get mass $m_h^2 = 2\mu^2$ from the expansion of V around minimum.

The gauge boson A_μ get mass $m_A^2 = (ev)^2$ from the Higgs effect.

Notice also that Goldstone boson ϕ get the mass $\xi(ev)^2$, but this mass depends on gauge fixing. This should hint to us that **Goldstone field is unphysical (fictitious)**. Indeed, due to inhomogeneous term in the gauge transformatione rule (**??**) of ϕ, one can get rid this field.

Now one needs to introduce Faddeev-Popov ghosts calculating $\frac{\delta G}{\delta \alpha}$:

$$\frac{\delta G}{\delta \alpha} = \frac{1}{\sqrt{\xi}}(-\frac{1}{e}(\partial)^2 - \xi ev(v+h)) \Rightarrow$$

$$det(\frac{\delta G}{\delta \alpha}) =$$

$$\int [Dc][D\bar{c}] \exp[-\imath \int d^4x \bar{c}(x) \frac{1}{e\sqrt{\xi}}(-(\partial)^2 - \xi m_A^2(1+\frac{h}{v}))c(x)] \tag{37}$$

Note that although we are dealing with an abelian gauge theory the ghosts fields can not be ignored because of they interract to the Higgs field.

2.3. Propagators and their poles.

Now we can extract the propagators for all the fields from (**??**), (**??**).

The propagator for $A_\mu(x)$ is the inverse to the operator

$$g^{\mu\nu}k^2 - (1-\frac{1}{\xi})k^\mu k^\nu - m_A^2 g^{\mu\nu} = g^{\mu\nu}(k^2 - m_A^2) - (1-\frac{1}{\xi})k^\mu k^\nu \tag{38}$$

Hence

$$< A_\nu(k) A_\lambda(-k) >= \frac{-\imath}{k^2 - m_A^2}(g_{\nu\lambda} - (1-\xi)\frac{k_\nu k_\lambda}{k^2 - \xi m_A^2}) =$$

$$\frac{-\imath}{k^2 - m_A^2}(g_{\nu\lambda} - \frac{k_\nu k_\lambda}{m_A^2}) + \frac{-\imath k_\nu k_\lambda}{m_A^2(k^2 - \xi m_A^2)} \tag{39}$$

Thus, we are almost back to the progataor (**??**).

Propagator for Higgs field is

$$< h(k)h(-k) >= \frac{\imath}{k^2 - m_h^2} \tag{40}$$

Propagator for Goldstone field is

$$< \phi(k)\phi(-k) >= \frac{\imath}{k^2 - \xi m_A^2} \tag{41}$$

Propagator for gosts c field is

$$< c(k)c(-k) >= \frac{\imath}{k^2 - \xi m_A^2} \tag{42}$$

We see that poles for propagators A_μ and Goldstone ϕ are at the same place. It will lead to concelations in all physical amplitudes in such a way that the result will not be dependent on ξ.

In other words, **one can show that Green's functions and S-matrix of gauge invariant operators will not depend on ξ**.

This statement can be proved by BRST symmetry but we will show this in some particular example.

2.4. Renormalizability and unitarirty.

Using the freedom to choose the value of ξ one can make two important observations.

1. $\xi = 0$.

In this case all the propagators fall as $\frac{1}{k^2}$. So that one can apply the standard analysis of diagrams divergences to conclude that **the theory is renormalizable**.

2. $\xi = \infty$.

Then

$$\begin{aligned} < A_\nu(k)A_\lambda(-k) >= \frac{-\imath}{k^2 - m_A^2}(g_{\nu\lambda} - \frac{k_\nu k_\lambda}{m_A^2}) \\ < \phi(k)\phi(-k) >= 0 \\ < c(k)c(-k) >= 0 \end{aligned} \tag{43}$$

Thus, the unphysical fields ϕ and c does not propagate. At the same time, the gauge field has on mass-shell only 3 space-like polarizations. Indeed, the tensor part of the propagator for A_μ is given by the sum over the polarizations:

$$\sum_{\epsilon\perp k}\epsilon^{\nu}\epsilon_{\mu}=\delta^{\nu}_{\mu}-\frac{k^{\nu}k_{\mu}}{m_A^2} \tag{44}$$

But in the own reference system of vector particle the right hand side of (**??**) is nothing else but the projection operator on the space-like directions. **Hence, the theory is unitary**.

2.5. Higgs effect for fermions.

Now we are going to demonstrate ξ-independence of amplitudes by an example.

Let us add the massless fermions and consider the Lagrangian

$$\begin{aligned} L(A,h,\phi,\psi) &= L(A,h,\phi)+ \\ \bar{\psi}_L(\imath\gamma^{\mu}D_{\mu})\psi_L+\bar{\psi}_R(\imath\gamma^{\mu}\partial_{\mu})\psi_R &-\lambda_f(\bar{\psi}_L\Phi\psi_R+\bar{\psi}_R\Phi^{*}\psi_L), \\ D_{\mu} &= \partial_{\mu}+\imath eA_{\mu} \end{aligned} \tag{45}$$

The gauge transformations for fermions are given by

$$\psi_L\to\exp\left(\imath\alpha(x)\right)\psi_L,\ \ \psi_R\to\psi_R \tag{46}$$

Notice that the left-chiral fermions $\psi_L(x)$ and right-chiral fermions $\psi_R(x)$ have different gauge transformation rules. The right-chiral fermions are singlets w.r.t. the gauge group $U(1)$ so they do not interract with the EM field, while the left-chiral fermions transform by the standard rules so that they interract to EM field. This left-right asymmetry is similar to that in the Standard Model of electro-week interractions. At the same time the fermions interract with the complex scalar field in sach a way to conserve the gauge invariance and get the masses by the Higgs mechanism. Thus, **the Lagrangian (??) is similar to the Standard-Model Lagrangian**.

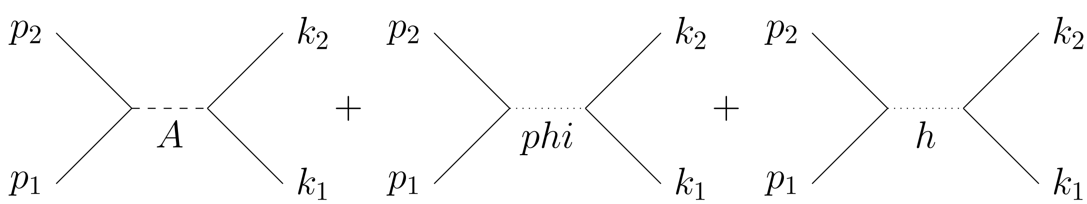
To see the Higgs effect for fermions we rewrite the interraction terms using the left-right chirality projectors $\frac{1\pm\gamma^5}{2}$:

$$\begin{aligned}
\lambda_f(\bar{\psi}_L\Phi\psi_R + \bar{\psi}_R\Phi^*\psi_L) = \\
\lambda_f(\Phi\bar{\psi}\frac{1+\gamma^5}{2}\psi + \Phi^*\bar{\psi}\frac{1-\gamma^5}{2}\psi) = \\
\frac{\lambda_f}{\sqrt{2}}((v+h(x)+\imath\phi(x))\bar{\psi}\frac{1+\gamma^5}{2}\psi + \\
(v+h(x)-\imath\phi(x))\bar{\psi}\frac{1-\gamma^5}{2}\psi) = \\
\frac{\lambda_f}{\sqrt{2}}((v+h(x))\bar{\psi}\psi + \imath\phi(x)\bar{\psi}\gamma^5\psi) = \\
m_f\bar{\psi}\psi + \frac{\lambda_f}{\sqrt{2}}(h(x)\bar{\psi}\psi + \imath\phi(x)\bar{\psi}\gamma^5\psi), \\
m_f = \frac{\lambda_f v}{\sqrt{2}}
\end{aligned} \tag{47}$$

Thus, the fermions become massive.

2.6. Green's functions do not depend on gauge fixing: Example.

We want to calculate main contribution to the fermion-fermion scattering amplitude in this theory. It is given by three diagrams:



(48)

In the first diagram the fermions exchange by the gauge field A_μ. According to the Feinmann rules this amplitude is given by

$$\imath M_A = (-\imath e)^2 \bar{u}(p_2)\gamma_\mu \frac{1-\gamma^5}{2} u(p_1)$$

$$\frac{-\imath}{q^2 - m_A^2}(g^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2 - \xi m_A^2})\bar{u}(k_2)\gamma_\nu \frac{1-\gamma^5}{2} u(k_1) =$$

$$(-\imath e)^2 \bar{u}(p_2)\gamma_\mu \frac{1-\gamma^5}{2} u(p_1)$$

$$(\frac{-\imath}{q^2 - m_A^2}(g^{\mu\nu} - \frac{q^\mu q^\mu}{m_A^2}) + \frac{-\imath q^\mu q^\mu}{m_A^2(q^2 - \xi m_A^2)})\bar{u}(k_2)\gamma_\nu \frac{1-\gamma^5}{2} u(k_1) =$$

$$(-\imath e)^2 \bar{u}(p_2)\gamma_\mu \frac{1-\gamma^5}{2} u(p_1) \frac{-\imath}{q^2 - m_A^2}(g^{\mu\nu} - \frac{q^\mu q^\mu}{m_A^2})\bar{u}(k_2)\gamma_\nu \frac{1-\gamma^5}{2} u(k_1) +$$

$$(-\imath e)^2 \bar{u}(p_2)\gamma_\mu \frac{1-\gamma^5}{2} u(p_1) \frac{-\imath q^\mu q^\mu}{m_A^2(q^2 - \xi m_A^2)} \bar{u}(k_2)\gamma_\nu \frac{1-\gamma^5}{2} u(k_1) \quad (49)$$

Now we rewrite the second term using the momentum conservation low $q = p_1 - p_2$ and Dirac equation of motion $p^\mu \gamma_\mu u(p) = m_f u(p)$:

$$\bar{u}(p_2) q^\mu \gamma_\mu \frac{1-\gamma^5}{2} u(p_1) =$$

$$\frac{1}{2}\bar{u}(p_2)((p_1 - p_2)^\mu \gamma_\mu - (p_1 - p_2)^\mu \gamma_\mu \gamma^5) u(p_1) =$$

$$\frac{1}{2}\bar{u}(p_2)(p_1 - p_2)^\mu \gamma_\mu u(p_1) + \frac{1}{2}\bar{u}(p_2)(\gamma^5 \gamma^\mu (p_1)_\mu + (p_2)_\mu \gamma^\mu \gamma^5) u(p_1) =$$

$$\frac{1}{2}\bar{u}(p_2)(m_f - m_f) u(p_1) +$$

$$\frac{1}{2}\bar{u}(p_2)(m_f \gamma^5 + m_f \gamma^5) u(p_1) =$$

$$m_f \bar{u}(p_2)\gamma^5 u(p_1) \quad (50)$$

Hence, the amplitude takes the form

$$(-\imath e)^2 \bar{u}(p_2)\gamma_\mu \frac{1-\gamma^5}{2} u(p_1) \frac{-\imath}{q^2 - m_A^2}(g^{\mu\nu} - \frac{q^\mu q^\mu}{m_A^2})\bar{u}(k_2)\gamma_\nu \frac{1-\gamma^5}{2} u(k_1) +$$

$$\frac{\lambda_f^2}{2}\bar{u}(p_2)\gamma^5 u(p_1) \frac{-\imath}{q^2 - \xi m_A^2} \bar{u}(k_2)\gamma^5 u(k_1) \quad (51)$$

In the second diagram the fermions exchange by the Goldstone field ϕ.

According to the Feinmann rules this amplitude is given by

$$\imath M_\phi = \frac{\lambda_f^2}{2}\bar{u}(p_2)\gamma^5 u(p_1)\frac{\imath}{q^2-\xi m_A^2}\bar{u}(k_2)\gamma^5 u(k_1) \tag{52}$$

Therefore

$$\imath M_A + \imath M_\phi =$$
$$(-\imath e)^2\bar{u}(p_2)\gamma_\mu\frac{1-\gamma^5}{2}u(p_1)\frac{-\imath}{q^2-m_A^2}(g^{\mu\nu}-\frac{q^\mu q^\mu}{m_A^2})\bar{u}(k_2)\gamma_\nu\frac{1-\gamma^5}{2}u(k_1) \tag{53}$$

so the result does not depend on ξ.

In the third diagram the fermions exchange by the Higgs field h so this amplitude does not depend on ξ. Thus **we have shown the ξ-dependent contributions mutual cancellation in the total fermion scattering amplitude**.

Appendix A. Ward identity for polarization operator in QED.

Let us consider the correlation function

$$I[J^\nu(y)] = \int [DA][D\psi][D\bar{\psi}]e\bar{\psi}(y)\gamma^\nu\psi(y)\exp{[\imath\int d^4x L_{QED}]} \tag{54}$$

Because of the measure $[D\psi][D\bar{\psi}]$ in (**??**) is invariant under the changes of variables

$$\psi(x)\to\acute{\psi}=\psi(x)+\imath e\alpha(x)\psi(x),\ \bar{\psi}(x)\to\acute{\bar{\psi}}(x)=\bar{\psi}(x)-\imath e\alpha(x)\bar{\psi}(x) \tag{55}$$

where $\alpha(x)$ is a small variation and the current is also invariant

$$\bar{\psi}(y)\gamma^\nu\psi(y)\to\bar{\psi}(y)\gamma^\nu\psi(y) \tag{56}$$

the paths integral (**??**) is unchanged:

$$\int [DA][D\psi][D\bar{\psi}]e\bar{\psi}(y)\gamma^\nu\psi(y)\exp{[\imath\int d^4x L_{QED}]} =$$
$$\int [DA][D\acute{\psi}][D\acute{\bar{\psi}}]e\acute{\bar{\psi}}(y)\gamma^\nu\acute{\psi}(y)\exp{[\imath\int d^4x L_{QED}]} \tag{57}$$

From the other hand $L_{QED} \to L_{QED} - e(\partial_\mu \alpha)(x)\bar{\psi}(x)\gamma^\mu \psi(x)$, therefore

$$0 = \delta_\alpha I[J^\nu] = \int [DA][D\psi][D\bar{\psi}](-\imath \int d^4x \partial_\mu \alpha(x) J^\mu(x) J^\nu(y)) \exp[\imath \int d^4u L_{QED}] \Rightarrow \frac{\delta_\alpha I[J^\nu]}{Z} = \imath \partial_\mu < \Omega|TJ^\mu(x)J^\nu(y)|\Omega >= 0 \quad (58)$$

Then making the Fourier transform and taken into account the translation invariance the identity (**??**) takes the form

$$k_\mu P^{\mu\nu}(k) = \delta(k_1 + k_2) k_{1\mu} P^{\mu\nu}(k_1, k_2) = \int d^4x d^4y (k_1)_\mu \exp[-\imath(k_1 x + k_2 y)] < \Omega|TJ^\mu(x)J^\nu(y)|\Omega >= \imath \int d^4x d^4y (\frac{\partial}{\partial x^\mu} \exp[-\imath(k_1 x + k_2 y)]) < \Omega|TJ^\mu(x)J^\nu(y)|\Omega >= -\imath \int d^4x d^4y \exp[-\imath(k_1 x + k_2 y)] \frac{\partial}{\partial x^\mu} < \Omega|TJ^\mu(x)J^\nu(y)|\Omega >= 0 \quad (59)$$

Or

$$\partial_\mu < \Omega|TJ^\mu(x)J^\nu(y)|\Omega >= \int d^4k_1 d^4k_2 \imath (k_1)_\mu P^{\mu\nu}(k_1, k_2) \exp[\imath k_1 x + \imath k_2 y] = 0 \quad (60)$$